

IMPACTS OF WAVE CLIMATE WITH BI-MODAL WAVE PERIOD ON THE PROFILE RESPONSE OF GRAVEL BEACHES.

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Abstract: Gravel beach recharge design and assessment tools are based upon wave conditions defined by integrated parameters that assume constant spectral shape. Field observations on the English Channel Coast show that the beach response to wave conditions, characterised by spectra with bi-modal wave periods, may result in significantly greater overwashing or crest erosion than the simple parametric models would suggest. The observations suggest the need to redefine design conditions by reference to bi-modal conditions for those sites that are subject to both swell and wind waves.

Introduction

Gravel beaches are widespread along the English Channel coast. They are characterized by sediments with high permeability, which provides the basis of effective wave energy dissipation. Because of their efficiency at dissipating wave energy within a relatively low volume of sediment, gravel beaches often form an integral part of the coastal defence system, used to protect against overtopping, overwashing or structure toe scour. Fringing gravel beaches may provide protection in combination with seawalls or other structures, whilst gravel barrier beaches may provide the sole protection at other sites.

Gravel beach management may frequently include the introduction of beach recharge or other management efforts to maintain a minimum defined beach cross section. In general, the sites are expected to deliver a defined standard of protection related to various combinations of wave and water level conditions; these are usually defined by reference to a joint probability, related to a return period expectation. The return periods specified are invariably based upon the key characteristics of integrated wave parameters and extreme water levels. The standard of protection relates to pre-determined performance, typically related to defined overtopping discharge, resistance to overwashing, or undermining of the structures at the top of the beach.

The design of gravel beach recharge schemes, or the assessment of the status of current beaches, is usually developed using a variety of models; these may include numerical, physical and empirical approaches, which are typically analysed on the

basis of the anticipated performance of the beach under a variety of defined extreme near-shore conditions. The majority of hydrodynamic inputs used in the models (e.g. Powell, 1990; Bradbury, 2000) are based on simple integrated parameters of significant wave height (H_s), zero crossing wave period (T_z), peak spectral wave period (T_p), direction and (at the most sophisticated) a standardised spectral shape typical of the site. In all of the above cases the empirical frameworks are based upon results from physical model tests conducted using wave climate conditions with a simple spectral shape, characteristic of the JONSWAP spectrum. The relationship between T_z and T_p remains constant with constant spectral shape, and in this case $T_p=1.2T_z$. Application of the predictive frameworks is used typically to identify:

- a) the dynamic equilibrium shape of the beach profile based on the predicted response, for defined integrated parameter storm wave and water level conditions. In particular, the elevation and position of the wave run-up crest are considered to be key performance variables when designing beach recharge solutions or assessing beach performance.
- b) the possibility of overwashing or a breach arising from wave run-up exceeding the crest on a barrier beach. .

The beach manager needs to be able to estimate how the beach will respond under design conditions and whether the required standard of protection is adequate to achieve these aspirations.

Observations of beach performance at a number of sites on the English Channel Coast have suggested that the response of gravel beaches may not always reflect the suggested response of the empirical predictive tools currently used in design and assessment. Regrettably these observations have infrequently been supported by measurements of beach responses. Hydrodynamic measurements of tidal and nearshore conditions are regularly available. This paper examines the application of the theoretical techniques to gravel beach recharge design in the U.K. and highlights limitations, related particularly to the method of description of the wave climate and the associated beach profile response. The results draw on an extensive set of physical model data and also 20 years of field monitoring, at sites in southern England, where the theoretical techniques have been applied. The paper focuses in particular on beach responses of the gravel barrier at Hurst Spit and at the beaches of Milford-on-Sea and at Hayling Island. All sites are in Hampshire, UK.

Experimental development of current predictive parametric frameworks

Advances in predictive techniques for the management of gravel beaches have largely been confined to parametric predictive models developed from small-scale model tests (Powell, 1990; Bradbury, 2000). More recently, physics based numerical models have been developed (Williams et al 2010) but these have rarely been tested for the practical applications of beach management. Both approaches require field validation.

Development of the empirical parametric frameworks examined within this paper is originally based on physical model testing at scales ranging from 1:20 to 1:40. There are two main approaches to scaling of mobile beach sediment in physical models. The first uses Froudean similitude when the material is scaled geometrically; this typically results in a reduced model permeability and a consequent flattening of profile response, relative to full scale. The alternative approach is based upon a technique originally developed by Yalin (1963), in which lightweight sediments with distorted geometry are used to represent the sediment. The theoretical techniques provide a modelling approach based upon independent solution of three key performance criteria. Beach slope is governed by permeability, and indirectly by grain size. A method of scaling of gravel beaches, which allowed both the correct permeability and drag forces to be reproduced in the model, has been described (Yalin, 1963a). This approach suggests that the percolation slope must be identical, in both model and prototype, to ensure that the permeability is reproduced correctly, in an undistorted model. Solution of each of the criteria results in conflicting results for the density and size grading of the model beach material for any scale, except unity, and the modelling solution is therefore an approximation. When combined, some relaxation of the rules is required to arrive at a practical, but theoretically flawed, modelling solution. Permeability is undoubtedly extremely important and controls internal flow within the beach. The limited range of lightweight materials presents the physical modeller with a series of compromises when modelling gravel at small scale. Some questions have been raised about the validity of the lightweight modelling approach, but the empirical models derived from this technique (Powell, 1990; Bradbury, 2000) are widely used in the design and management of gravel beaches. Particular concerns have been expressed at the rate of evolution of the dynamic equilibrium profile of the beach, wave run-up and also the evolution of the key beach descriptors, such as the crest, the step and the base of the profile.

Powell's parametric model provides a predictive framework based upon a series of empirically derived dimensionless equations used to identify a defined set of beach profile descriptors. By contrast, Bradbury (2000) provides a simple assessment of the likelihood of overwashing on gravel barrier beaches. Although some limited

earlier attempts have been made to validate this modelling approach in field investigations, the complexity of installation of field instrumentation and the lack of ability to control conditions within a systematic field observation framework mean that large scale modelling provides the best possibility for the investigation of the morphodynamic response of these complex systems.

Large scale model validation

Two independent series of large scale model tests (Blanco et al, 2002; Williams et al 2009)) have provided suitable data to test Powell's (1990) framework. No large scale test programme has been suitable to assess Bradbury's (2000) barrier inertia framework in detail, although a very limited proportion of Williams et al (2009) test data provides some useful comparison.

Observations made by Blanco et al (2002) in large scale tests indicate that the crest evolution occurs very rapidly on a gravel beach. A comparison between the full-scale gravel test data and Powell's (1990) parametric framework suggests generally similar results (Bradbury and McCabe, 2003). The crest-berm approaches a dynamic equilibrium elevation after approximately 3000 waves, at a defined water level, although this does continue to grow slowly. A significant proportion of the profile evolution occurs after a period of 500-1000 waves. The full scale test results suggest that the run-up crest elevation may be slightly higher (typically 0.1-0.2m) than Powell's (1990) framework indicates for comparable conditions (Bradbury and McCabe, 2003). The position of the crest shows remarkable consistency with the small scale observations. On the basis of these comparisons, it appears that the small scale derived empirical predictive framework provides an adequate description of the beach response, at least for the beach crest variables. Comparisons of other predictive variables indicate that certain of the parameters are better represented than others. In particular, the gravel profile- base and step are less well reproduced than the crest. These variables are highly susceptible to the last few waves prior to measurement; this observation is consistent with those made by Powell (1990).

By comparison, Williams (2009) full scale testing programme also suggests that Powell (1990) under-predicts the crest-berm elevation. Observed differences are similarly small (typically 0.1m for comparable conditions). The position of the crest is less well defined however, tending to form further to landwards than predicted. The width of the active beach between the crest and step positions, measured in the full scale tests, is notably larger (several m) than is suggested by Powell (1990). A significant difference in test variables is related to the gravel grain size grading, which was somewhat smaller ($D_{50}=10\text{mm}$) in Williams (op cit) programme, relative to Blanco (2002) ($D_{50}=21\text{mm}$); this may provide some explanation for the difference in the width of the active profile, which may reflect lower beach

permeability in Williams' programme.

By contrast to the gravel model tests, the response of a mixed sand and gravel beach presents quite different profile response to the gravel beach (Bradbury and McCabe, 2003). The small quantity of tests makes conclusive observation difficult to ascertain. It is clear that the crest elevation is likely to be lower and the location further to landwards, on the mixed beach than the gravel beach, for comparable hydrodynamic conditions; this has major design implications for beach recharge solutions. It is clear that further development of empirical predictive solutions is required for mixed beaches. Currently some designers revert to tools developed for gravel beaches to aid the design of mixed beach recharge schemes. This is undesirable, as it is clear that such application of gravel models to a mixed beach situation will result in under-prediction of the location of the crest position relative to still water level, and will consequently result in a smaller safe cross section.

On the basis of the independent large scale observations, it is suggested that the empirical frameworks do provide a suitable starting point for prediction of the profile response of gravel beaches, although some limitations are immediately noted.

Design applications and aspirations

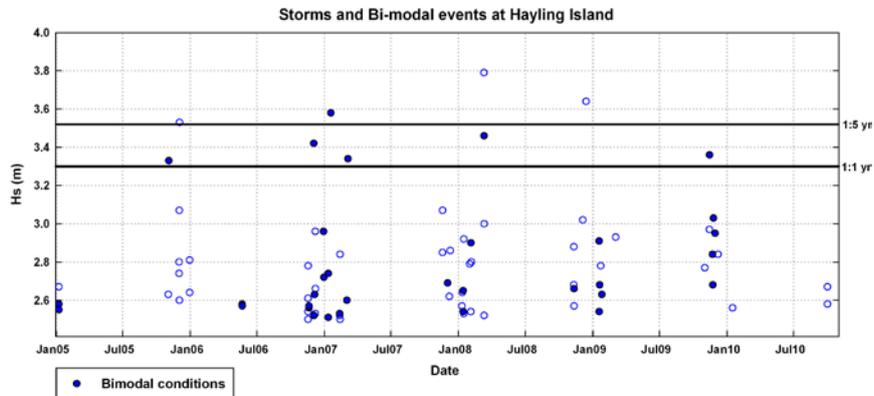
Powell's empirical framework is often used in the design of beach recharge schemes. The typical approach used is to assess the required beach cross section to enable a dynamic equilibrium profile to form, under the design wave and water level conditions. An allowance is usually made for cross-shore losses over time, to determine the potential scheme life, often in conjunction with beach plan shape modelling. If an inadequate volume of material is available for the dynamic equilibrium profile to form, for given hydrodynamic conditions, the empirical framework will not predict a dynamic equilibrium profile; this reflects the cross shore conservation of volume assumed.

Bradbury's empirical framework is often used to assess the vulnerability of barrier beaches to overwashing and in the design of barrier beach recharge schemes. The approach requires assessment of the surface emergent cross-section area of the barrier at the storm peak water level, relative to the incident wave conditions. Threshold curves based on the parametric framework provide a simple method to assess whether overwashing is likely under the defined conditions.

Observations of notable events on the English Channel Coast

Several storm events have resulted in unexpected responses, during which the

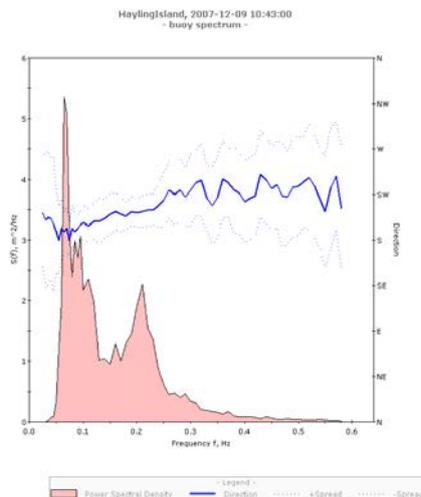
profile response of beaches and overtopping discharge has been significantly



different to that anticipated. Widespread overtopping occurred along the English Channel Coast during the storm event of 3 November 2005, including sites at Worthing and Hayling Island. The integrated parameter conditions for this event (Figure 1) suggest a return period of 1:1 years, yet no overtopping occurred during an event with a return period of 1:5 years a few weeks later. Overtopping was noted during 3 November 2005 for barrier beaches at Chesil Beach, Hurst Spit and Medmerry amongst others, when this was not expected.

Figure 1 Distribution of uni-modal and bi-modal storm events at Hayling Island wave buoy.

A number of similar events have been identified, where relatively frequently occurring integrated parameter conditions have resulted in much larger changes to the beach profile than expected.



Measured field responses for these events are limited and are generally confined to photographic records, although high quality wave and tidal data are available in support of the analysis of hydrodynamic conditions. The common feature of these damaging events is that wave-spectra are characterised by bi-modal wave periods, with both swell and wind wave energy peaks (Figure 2).

Figure 2 Bimodal spectrum at Hayling Island wave buoy

Field validation – practical considerations

Validation of both of the parametric frameworks is non-trivial in the field. Firstly, both are based upon the assumption that there is no net cross-shore loss of material, which is often not the case. Allowance can be made for such losses in application of the parametric frameworks.

Whilst it is a simple tool, Powell's (1990) empirical framework is particularly difficult to evaluate in the field. Most of the parametric variables in the formulae are submerged during storm conditions and cannot be measured directly. Because the response of the beach is dynamic, changes occur following the storm peak as the storm decays, thereby modifying the submerged profile formed at the storm peak. This essentially has the effect of shortening the active profile during the storm decay, rapidly modifying the location of the breaking point and wave base. Even under controlled laboratory conditions the position and elevation of the profile step is volatile and varies rapidly. In practical terms it is possible only to validate the two permanently surface emergent variables h_c and p_c , as modified by the storm peak. It is considered that, for the purposes of practical management, these are the most valuable measures of performance. These variables are included in criteria used most frequently by engineers to examine the beach performance and to assess the need to intervene on the beach.

The crest height should theoretically always form at a constant elevation proportional to the storm peak water level and wave conditions, provided that there is adequate material for a dynamic equilibrium profile to develop. The position of the crest should similarly form at constant proportional distance from the beach static water level intersection, depending on hydrodynamic conditions, although this is affected by mass balance calculations which reflect the pre-storm profile and the influence of oblique wave attack. These coordinate positions are relatively easy to measure following the storm, since they will be unaffected by post storm decay. They are usually identified by either a strand line or a distinct beach crest berm. On some occasions the crest position may be indistinct and surveyed locations may sometimes be incorrectly located; this is frequently a problem on recharged beaches where run-up occurs part way up a filled face.

In some instances a dynamic equilibrium profile is unable to form, due to the lack of available sediment within the beach cross-section. This is notable during particularly extreme or severe conditions, when overtopping may occur as a result. Such conditions were evident during the storm event of 3/11/2005, at many sites on

the English Channel Coast, although predictions suggested that sufficient material was available for the dynamic equilibrium profile to develop.

By contrast, Bradbury's simple empirical framework is more easily assessed in the field. As the framework provides a simple prediction of whether certain conditions are likely to cause overwashing or not, the response is relatively easily measured. Clear indicators of overwashing are always evident on the lee face of the beach following such conditions. Such responses are characterized by crest elevation lowering, washover fans, veneer deposits of sediment on the lee face, or scoured gulleys on steeper engineered slopes.

Comparisons between field responses and Powell's parametric framework

The two parametric profile descriptor variables (p_c , h_c) are examined by reference to measured beach response, tidal and wave data (Table 1).

p_c = crest-position relative to storm peak water level and beach intercept (m)
 h_c = crest elevation- relative to storm peak water level (m)

The dimensionless form of the multivariable formulation (Powell, 1990) is shown below for the two crest parameter variables (p_c , h_c).

$$p_c D_{50} / H_s L_m = -0.23 \left(H_s T_m g^{1/2} / D_{50}^{3/2} \right)^{-0.588} \quad (1)$$

where: D_{50} = sediment size (m); H_s = significant wave height (m), L_m = wave length (m), T_m = mean wave period (s).

$$h_c / H_s = 2.86 - 62.69 (H_s / L_m) + 443.29 (H_s / L_m)^2 \quad (2)$$

Valid range for both formulae = $0.01 < H_s / L_m < 0.06$

Wave conditions vary typically throughout the course of the tidal cycle and the theoretical beach crest elevation and position responds to this variability. Calculations of the most damaging condition for each event i.e. the highest and furthest displaced crest, occurred at the highest water level in every case (Table 1), suggesting that water level is an overriding control on theoretical profile response at the sites tested. The most damaging condition over the course of the storm event was used for assessment of the theoretical crest descriptors for that storm.

Whilst the actual tidal elevation is variable, the referencing method used by Powell *op cit.* relates the crest position and relative elevation to static water level at the time of the storm peak. The influence of tidal elevation is removed from the analysis therefore and the crest elevation (h_c) is a useful indicator of the beach crest solely to wave conditions. The parametric framework therefore provides a means of isolating the beach response to wave conditions. The two formulae are examined in dimensioned form in context with the wave variables (Figures 3 and 4). Sensitivity testing of the formulation suggests that the influence of D_{50} on the coordinates of the two crest variables is very small, although it should also be noted that the range of grain size variables actually tested was very limited. This is certainly worthy of future investigation.

Table 1 Range of wave and tidal conditions tested in this investigation.

Hayling 14/01/2005 Post Storm	Storm Date	SWL (ODN)	H_s (m)	T_z (s)	T_p (s)	Spectral shape
Storm Peak- Highest Wave Height	08/01/2005	0.44	2.7	5.3	8.3	
Storm Peak- Highest Water Level	08/01/2005	1.76	2.6	5.6	13.3	bi-modal
Hayling 04/11/2005 Post Storm						
Storm Peak- Highest Wave Height	03/11/2005	2.04	3.3	6.9	18.2	
Storm Peak- Highest Water Level	03/11/2005	2.44	3.3	6.7	15.4	bi-modal
Hayling 10/12/2007 Post Storm						
Storm Peak- Highest Wave Height	09/12/2007	0.71	2.9	7.0	13.3	
Storm Peak- Highest Water Level	09/12/2007	2.25	2.6	5.6	15.4	bi-modal
Hayling 05/12/2008 Post Storm						
Highest Hs	04/12/2008	-0.23	3.0	5.8	7.7	
Highest Water Level	04/12/2008	1.72	1.5	4.1	5.0	uni-modal
Hayling 09/11/2010 Post Storm						
Highest Hs	08/11/2010	1.05	3.2	6.0	8.3	
Highest Water Level	08/11/2010	2.22	2.4	6.3	8.3	uni-modal
Milford 04/10/2010 Post Storm						
Highest Hs	01/10/2010	0.13	2.3	5.3	7.7	
Highest Water Level	01/10/2010	0.70	2.0	5.0	7.7	uni-modal

*Wave conditions measured in 10-12m water depth

Data is scattered to both sides of the predicted elevation for the incident storm conditions (Figure 3). All observed data connected with the response to uni-modal storm events lies within limits of ± 1.5 m of the predicted crest elevation relative to the storm static water level elevation. By contrast, the differences between predicted and measured data for bi-modal conditions are wider spread. Measured crest elevations for the bi-modal events are consistently higher than the parametric

predictions suggest, although all data lies within 2m of the parametric predictions. No obvious trend is evident and data do not generally fit the predictive model particularly well. There is no obvious visible trend within this data set that differentiates the bi-modal data from the uni-modal data. Conditions during the event of 4/12/2008 are close to the threshold criteria for definition of bi-modal conditions (Mason et al 2008) with the swell energy component reaching ~20%. This reflects also on the response of crest position (Figure 4). The apparent under-prediction of the crest elevation for the event of 8/11/2010 may be a function of oblique wave attack, which may reduce run-up. Incident conditions approached at $\sim 20^\circ$ angle for this event whilst all other events approximated to normally incident.

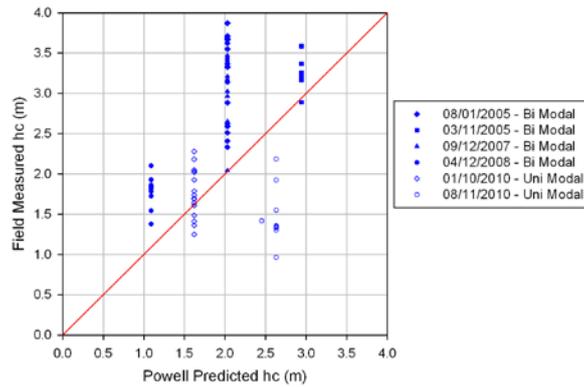


Figure 3 Comparison of measured and predicted crest elevation

Observations of the crest position (p_c) suggest that the empirical framework may typically under-predict the crest position, for both bi-modal and uni-modal conditions (Figure 4). All of the uni-modal data lies within 10m of the predicted position, and some observations agree very well with the predicted crest position, although the data is too scattered for a good correlation to be derived.

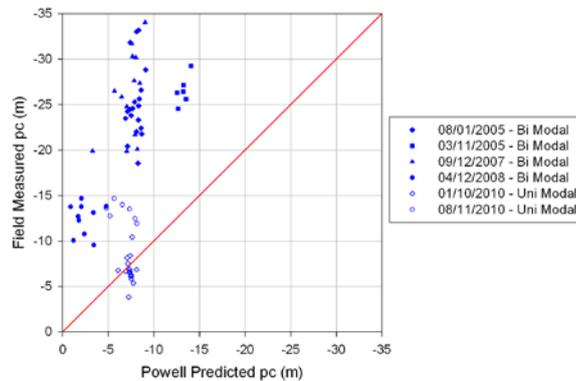


Figure 4. Comparison of measured and predicted crest position

Most of the observations made for bi-modal conditions are >10m landward of the predicted position however, and the measured crest position may be as much as 25m further landwards than predicted. This is highly significant for beach management.

There is a notable difference of the crest position between bi-modal and uni-modal events, for similar integrated parameter wave conditions. Both data sets for uni-modal and bi-modal conditions span a similar range of wave conditions (Table 1) and so the responses should be expected to be similar. The delay between the pre-storm and post storm surveys may possibly account for some of the observed differences. It is notable that the best fit data is associated with closely spaced pre and post storm surveys, when cross shore losses are less likely to be significant.

Comparisons between field responses and barrier inertia framework

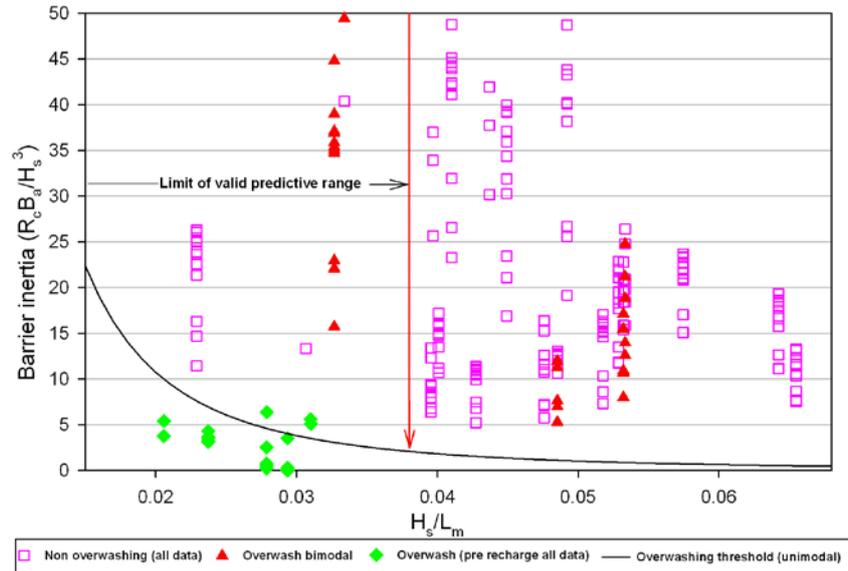
The barrier inertia parameter forms part of a relatively simple parametric framework which provides a quick indication of whether a gravel barrier beach is likely to undergo overwashing during defined condition (Bradbury, 2000). A basic empirical framework has been derived relating the barrier inertia parameter ($R_c B_a / H_s^3$) to the wave steepness parameter (H_s / L_m) and with a derived overwashing threshold (Figure 5), based on the barrier geometry. Where:

R_c = freeboard from storm peak water level to barrier crest;

B_a = cross section area of surface emergent barrier above storm peak water level

The parametric overwashing threshold curve is examined by reference to measured beach response, tidal and wave data for storms at Hurst Spit (Figure 5) dating back to 1989. The vast majority of data follows a large scale beach recharge scheme, which was implemented in 1996. The scheme has been designed on the basis of extensive 3-dimensional wave basin testing under random wave conditions. Tests have been used to determine the appropriate cross section of gravel recharge, to avoid overwashing in all but the most extreme conditions, and to identify critical conditions that could be used as a guide to inform the need for intervention during long-term management. Wave conditions in the model studies were based on simple spectral shape (JONSWAP), so the design does not consider the consequence of bi-modal conditions. As the scheme was designed to limit overwashing, there should be little expectation of post recharge overwashing events. Supplementary data is provided for pre-recharge conditions, when overwash occurred on a more regular basis, on a barrier of more natural form. Note that spectral data is not available for these earlier (pre-1996) events.

A total of 22 post recharge events have been analysed. The data demonstrates that the barrier inertia parameter has performed reliably under uni-modal conditions, with no overwashing events noted, as expected. Overwashing conditions are also identified reliably for pre beach recharge conditions, very close to the predictive



threshold. Three events have resulted in overwashing, for conditions which are not predicted by the current parametric framework. Each of these events is characterised by bi-modal wave conditions. The overwash across relatively large beach cross sections during these events suggests a significantly higher wave run-up during these conditions.

Figure 5 Comparisons between field measurements and the barrier inertia threshold.

Influence of bi-modal wave period on beach response

The field analysis of both parametric frameworks tested indicates that bi-modal wave conditions result in a different beach response than might be expected when testing using parametric frameworks, based on simple integrated wave parameters and uni-modal wave conditions. This is consistent with other laboratory observations of profile response (Coates et al, 1998). The common factor noted at all sites during the un-anticipated overtopping and overwashing events, is the presence of wave spectra characterised by bi-modal wave periods, with significant components of swell wave energy (>20%). In particular, run-up elevations are higher, cut back of beach crest berms is greater and overwashing is more frequent

under bi-modal wave conditions than theoretical design formulae might suggest.

Conditions characterised by bi-modal wave periods were not considered at the design stage for any of these beach recharge projects. It is of some concern to designers of beach recharge schemes that bi-modal wave conditions may result in more overwashing, or considerably increased erosion of the beach crest than might be predicted using conventional design tools.

Isolation of variables is particularly difficult during the storm events since the sequence and intensity of conditions varies during the course of the storm event. In particular, the proportion of swell energy varies significantly through the events; this is considered to be of particular significance. Detailed examination of wave climate conditions associated with these events has identified that a significant proportion of the energy component (20-40%) has typically been in the swell energy range of frequencies.

The temporal and spatial distribution of bi-modal events is significant within the western English Channel (Mason et al 2008). Figure 1 shows the temporal distribution of bi-modal and uni-modal events at Hayling Island, suggesting about 40-60% of storm events are bi-modal at this site; these trends are observed on a region wide basis, which provides more cause for concern.

Conclusion

Cross-shore profile responses of gravel beaches are not well described by empirical models used widely in beach recharge design, when subject to bi-modal wave period conditions; these occur regularly and are widespread on the south coast of the U.K. (Mason et al 2008). The models generally under-predict wave run-up and in particular crest-berm cut-back in such conditions, when simple integrated wave parameters (H_s , T_m) are used. Similarly the barrier inertia parameter may fail to predict overwashing under some bi-modal conditions. The barrier inertia parameter appears to work reasonably well under uni-modal conditions and Powell's (1990) framework provides results of variable reliability.

These observations suggest that spectral shape is a key variable that is not normally considered in the design process. The response of the beach under these conditions appears to be worse than conditions defined by spectra with simple shapes. This is consistent with other laboratory observations of profile response (Coates et al, 1998).

Current design guidance does not provide an obvious means of dealing with this design variable, apart from site specific physical model testing. Monitoring has

identified a need for a general review of the standard of service of beach management schemes and the need to redefine design conditions by reference to bi-modal wave conditions.

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